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Applying Singular Perturbation Theory for Reducing the Size of Dynamic Model of the Electronic Circuits

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ABSTRACT

The dynamic modelling of nonlinear electronic circuits often results in high-dimensional systems of differential equations that are computationally expensive to solve, particularly when incorporating parasitic elements with widely varying time scales. This research proposes a dimensionality reduction framework utilizing singular perturbation theory applied to the chaotic Chua circuit. By decomposing the system dynamics into "slow" (outer) and "fast" (inner) time scales, and invoking Tikhonov's theorem to validate the asymptotic correctness, a reduced-order model is derived. A uniform approximation is subsequently constructed by mathematically matching the boundary layer transients with the steady-state behaviour. Numerical simulations compare this approximation against the full system solved via standard ODE solvers, revealing that the uniform approximation achieves high fidelity with negligible absolute errors. The results confirm that singular perturbation is an effective technique for minimizing computational cost without compromising dynamical accuracy, presenting significant potential for scaling to higher-dimensional problems.

Keywords: Perturbation theory, Tikhonov's theorem, Chua circuit

INTRODUCTION

The dynamic modelling of electronic circuits is fundamental to understanding system behaviours, particularly when characterizing complex relationships between voltage, current, and nonlinear resistive elements. However, accurate dynamic modelling often results in high-dimensional systems of ordinary differential equations (ODEs) that are computationally expensive to solve, especially when integrating stiff equations with widely varying time scales. As noted in recent studies on spacecraft electrical systems and microgrid clusters, the presence of parasitic parameters—such as small inductances and capacitances—often creates multiple time scales that complicate numerical stability [1], [2]. To address this, singular perturbation theory (SPT) offers a robust mathematical framework, allowing researchers to decompose a high-dimensional system into reduced-order "slow" and "fast" subsystems. By mathematically isolating the boundary layer phenomena where fast transients occur, SPT provides a systematic method for model reduction that retains the essential dynamics of the system while significantly lowering computational overhead [3], [4].

METHODOLOGY

Mathematical Modelling

A1. Chua Circuit

The Chua circuit was first invented in 1983 by Leon O. Chua. It is a simple electronic circuit that exhibits chaos and many bifurcation phenomena. The existence of chaotic attractors from the Chua circuit had been confirmed numerically by Matsumoto via computer simulations, observed experimentally by Zhong and Ayrom in laboratory, and mathematically proved by Chua et al. in [5].

The circuit diagram of the Chua Circuit is shown in Figure 1. The circuit consists of five circuit elements: two capacitors C_1 and C_2, one inductor L_1, one linear resistor R and one nonlinear resistor N_R. The component N_R is a nonlinear negative resistance called a Chua's diode. It is usually made of a circuit containing an amplifier with positive feedback.

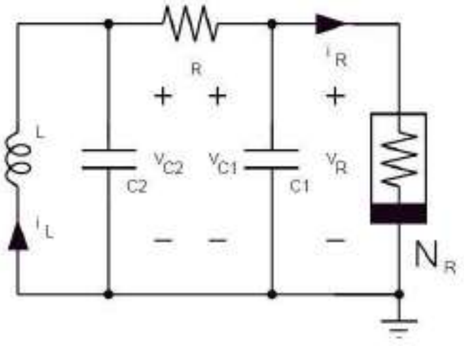

Figure 5.1: The Chua circuit

Recent literature highlights the evolving complexity of Chua’s circuit, extending beyond simple chaotic demonstration to sophisticated control and synchronization applications. Xu et al. demonstrated the utility of Field-Programmable Gate Arrays (FPGAs) in numerically simulating synchronized Chua circuits, emphasizing the need for efficient model representations to match hardware constraints [6]. Similarly, Sun et al. and Chaudhury et al. have explored synchronous dynamics in robotic arms and heterogeneous oscillators driven by Chua circuits, respectively, reinforcing the necessity of precise yet computationally manageable mathematical models [7], [8]. Furthermore, the investigation of Jacobi stability in Muthuswamy–Chua–Ginoux systems by Wang et al. illustrates that even in modified circuit topologies, the core challenges of nonlinear stability and dimensionality persist [9]. These studies collectively suggest that while the physical implementations of chaos are advancing, the demand for analytical methods that can simplify these high-dimensional interactions without losing topological accuracy is higher than ever.

A2. Mathematical Model of Chua Circuit

The Chua circuit can be analysed by using Kirchhoff's circuit laws, the dynamics of this circuit can be modelled by a system of three nonlinear ordinary differential equations (ODEs) in the variables $x(t)$, $y(t)$ and $z(t)$, which represent the voltages across the capacitors C_1 and C_2, and the intensity of the electrical current in the inductor L_1, respectively. The system of ODEs has the form

$$\frac{dx}{dt} = \frac{1}{\epsilon}\left(z - c_1x^3 - c_2x^2 - \mu\, x\right)$$

$$\frac{dy}{dt} = -\beta x \qquad (1)$$

$$\frac{dz}{dt} = -\alpha x + y + bz$$

Here the ϵ holds a small value while the parameters c_1, c_2, μ, β, a and b are determined by the particular values of the circuit components.

A3. Singular Perturbation Theory

Let consider a multiple time scales system of the form

$$\epsilon\frac{dx}{dt} = f(x,y), \quad x(0,\epsilon) = x_0 \qquad (2)$$

$$\frac{dy}{dt} = g(x,y), \quad y(0,\epsilon) = y_0$$

where $x \in R^n, y \in R^m, \epsilon \in (0,1], t \in [0,T], f$ and g are sufficiently smooth function in the domain

$$D = \{(x,y): |x| \le d_1, |y| \le d_2\}, \qquad d_1, d_2 \in R_+.$$

That is, $f \in C^r(D, R^n)$ and $g \in C^r(D, R^m)$ for $r \ge 2$. The variable x is called the fast variable while variable y is the slow variable in the system.

Formally, by setting $\epsilon = 0$, we obtain a differential algebraic system:

$$0 = f(\bar{x}, \bar{y}), \tag{3}$$

$$\frac{d\bar{y}}{dt} = g(\bar{x}, \bar{y}), \quad \bar{y}(0,0) = y_0$$

This system is called the "degenerate system" in the singular perturbation theory as its order is less than the order of system (2). Since system (3) has reduced to the differential algebraic equation (DAE) form, system (2) is an example of a singularly perturbed system. Then Tikhonov theorem [10] is referred as it shows how well the system (2) is approximated by the unperturbed system (3).

On the other hand, by using the scaled time variable $\tau = t/\epsilon$, system (2) can be reformulated in an equivalent form:

$$\frac{d\tilde{x}}{dt} = \tilde{f}(\tilde{x}, \tilde{y}), \quad \tilde{x}(0,\epsilon) = x_0 \tag{4}$$

$$\frac{d\tilde{y}}{dt} = \epsilon \tilde{g}(\tilde{x}, \tilde{y}), \quad \tilde{y}(0,\epsilon) = y_0$$

where $\epsilon \in (0,1], \tau \in \left[0, \frac{T}{\epsilon}\right]$, $\tilde{f}$ and $\tilde{g}$ are sufficiently smooth functions in the domain

$$\tilde{D} = \{(\tilde{x}, \tilde{y}): |\tilde{x}| \le c_1, |\tilde{y}| \le c_2\}, \qquad c_1, c_2 \in R_+.$$

This system is a regularly perturbed ODE. By referring to the regular perturbation theory, we know that the system (4) is well-approximated by the adjoined system [11].

In general, two solutions (approximations) can be derived for a singularly perturbed problem when setting $\epsilon = 0$. The "outer solution", the solution for the degenerate system (3), provides a good approximation outside the boundary layer. On the other hand, the "inner solution", the solution for the adjoined system (4), provides a good approximation in the boundary layer. A process of matching of these solutions is essential to relate the two solutions and obtain a complete solution that approximates the system dynamics throughout the whole problem domain. Kaplun and Langerstorm [12] employed "intermediate matching" which is based on the so-called "overlapped hypothesis", that is, an assumption that there exist extended domains of validity for the outer and inner expansions with a non-empty intersection, which is where both the outer and inner expansions are valid and can be matched. More precisely, the inner $\varphi_0(\tau)$ and outer approximation $\phi_0(t)$ are matched if they have a common limit as ϵ tends to zero, hence the requirement for matching can be represented as

$$\lim_{\epsilon \to 0} \phi_0(t) = \lim_{\epsilon \to 0} \varphi_0(\tau)$$

Subsequently, the final, matched solution that well-approximates the actual solution on the whole problem domain is called the "uniform approximation". This approximation can be obtained by adding the inner and outer solutions and subtracting their common limit as follows:

$$\Phi_u(t) = \phi_0(t) + \varphi(\tau) - \eta_c$$

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where $\Phi_u(t)$ is the uniform approximation and

$$\eta_c = \lim_{t\to 0} \phi_0(t) = \lim_{\tau\to\infty} \varphi_0(\tau)$$

is the common limit.

Simulation

B1. Outer solution

Within the slow time scale, consider the system of ODEs (1). By letting $\epsilon = 0$, the first equation of system (1) reduces to an algebraic equation

$$z - c_1 x_o^3 - c_2 x_o^2 - \mu\, x_o = 0 \tag{5}$$

Therefore, the voltages across the capacitors C_1, x_o, in the outer region, appears to be constant in the post-transient time course. Here, x_o , y_o and z_o are used to replace the x, y and z respectively to denote the x, y and z in outer region.

Meanwhile, rewrite the second and the third equations of the ODEs system (1) with the substitution of x_o gives

$$\frac{d\,y_o}{dt} = -\beta\, x_o \tag{6}$$

$$\frac{d\,z_o}{dt} = -\alpha x_o + y_o + b z_o$$

where x_o can be found numerically from the equation (5).

B2. Inner Solution

On the other hand, upon replacement of the scaling dimensionless variables, $\tau = \frac{t}{\epsilon}$ into the system of equations (1), the new governing equations can be written as

$$\frac{dx_I}{dt} = z - c_1 x_I^3 - c_2 x_I^2 - \mu x_I$$

$$\frac{dy_I}{dt} = -\epsilon\beta x_I \tag{7}$$

$$\frac{dz_I}{dt} = -\epsilon(\alpha x_I + y_I + b z_I)$$

Subscript I here indicates the inner solution.

If we set $\epsilon \to 0$ as expected in the perturbation theory, we will obtain $\frac{dy_I}{d\tau} = \frac{dz_I}{d\tau} = 0$. Hence, y_I and z_I are approximately constant throughout the system, that is, $y_I = y(0)$ and $z_I = z(0)$.

Subsequently, substitution of $z_I = z(0)$ into the first equation of system (7) leads to

$$\frac{dx_I}{dt} = z(0) - c_1 x_I^3 - c_2 x_I^2 - \mu x_I\ . \tag{8}$$

B3 Matching and Uniform Approximation

The inner solution which provides a good approximation in the transient period, together with the outer solution which provides a good approximation in the post-transient period, comprise a total solution for the system. These

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solutions have a common limit or overlap term, that is, where the outer solution begins to take over from inner solution. Hence matching is required here to get the uniform approximation.

By applying the matching condition to the voltages across the capacitors C_2 on y_I and y_o, we get

$$\lim_{t\to 0} y_o(t) = \lim_{\tau\to\infty} y_I(\tau) = \mathrm{y}(0).$$

While applying the matching condition to the intensity of the electrical current in the inductor L_1 on z_I and z_o gives

$$\lim_{t\to 0} z_o(t) = \lim_{\tau\to\infty} z_I(\tau) = \mathrm{z}(0).$$

On the other hand, imposing the matching requirement on the voltages across the capacitors C_1 gives

$$\lim_{t\to 0} x_o(t) = \lim_{\tau\to\infty} x_I(\tau) = \eta_c$$

Subsequently, we can have the uniform approximation x, y and z by adding the inner and outer solutions and subtracting their common limit, that is,

$$c_u = c_I + c_o - \eta_c$$

$$y_u = y_I + y_o - y(0) = y_o$$

$$z_u = z_I + z_o - z(0) = z_o$$

where these solutions well approximate the system dynamics throughout the whole problem domain.

RESULT DISCUSSION

We take the parameters $c_1 = \frac{44}{3},\ c_2 = \frac{41}{2}, \mu = 2, \beta = 1, a = 0.7, b = 0.24$ and $\epsilon = 0.05$ for the simulation of Chua circuit using singular perturbation theory.

Outer Solution

By considering the initial conditions $x(0) = 0, y(0) = 1, z(0) = 1$ and set $\epsilon = 0$, we obtain the outer approximation by solving the algebraic equations (5) and differential equations (6). The simulation was done based on the coding written via Matlab with a step size of 0.01 for $t \in [0.4]$. The results of the outer approximation are presented in Figure 2.

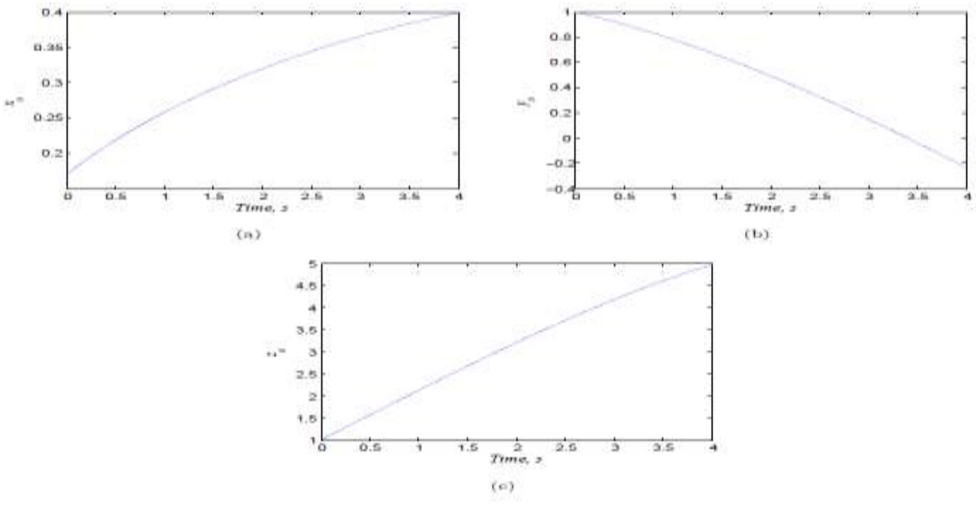

Figure 2: Outer approximations of x, y and z

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Inner Solution

On the other hand, we solve the equation (8) in the boundary layer to get the inner approximation. The simulation was done based on the coding written via Matlab with a step size of 0.001 for $t \in [0, 0.05]$, that is, we want to observe the rapid change in the boundary layer particularly. The result is represented pictorially in the Figure 3. We can see that the voltages across the capacitors C_1 at time $t = 0$ is equal to its initial value.

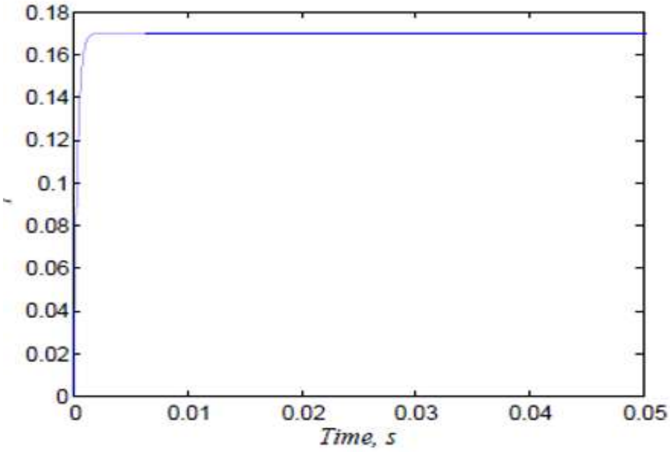

Figure 3: Inner approximation for x

An additional remark here is that we can see that both the scaled system in outer and inner approximations (in subsection B1 and B2) are equivalent to the original system (1). We have the small parameter ϵ appears in the equation $\frac{dx_O}{dt} = \frac{1}{\epsilon} f_1(x_O, y_O, z_O)$ of the outer solution, whereas ϵ appears in the equations $\frac{dy_I}{dt} = \epsilon f_2(x_I, y_I, z_I)$ and $\frac{dz_I}{dt} = \epsilon f_3(x_I, y_I, z_I)$. This enables the singular perturbation procedure to take place so that outer and inner approximations are obtained in different time scales.

Uniform approximation

Subsequently, we proceed to match the outer and inner solutions. More precisely, the voltages across the capacitors C_2 on y_I and y_o is

$$\lim_{t\to 0} y_o(t) = \lim_{\tau\to\infty} y_I(\tau) = \mathrm{y}(0) = 1.$$

While the intensity of the electrical current in the inductor L_1 on z_I and z_o are matched where

$$\lim_{t\to 0} z_o(t) = \lim_{\tau\to\infty} z_I(\tau) = \mathrm{z}(0) = 1.$$

On the other hand, the matching requirement is applied on the voltages across the capacitor C_1 and thus its common limit is $\lim_{t\to 0} x_o(t) = \lim_{\tau\to\infty} x_I(\tau) = 0.1696$ for this case.

As a consequence, the uniform approximations are for x, y and z are

$$x_u = x_I + x_o - 0.1696, \quad y_u = y_o\,, \quad z_u = z_o\,.$$

The uniform approximation for x, y and z are presented in Figure 4.

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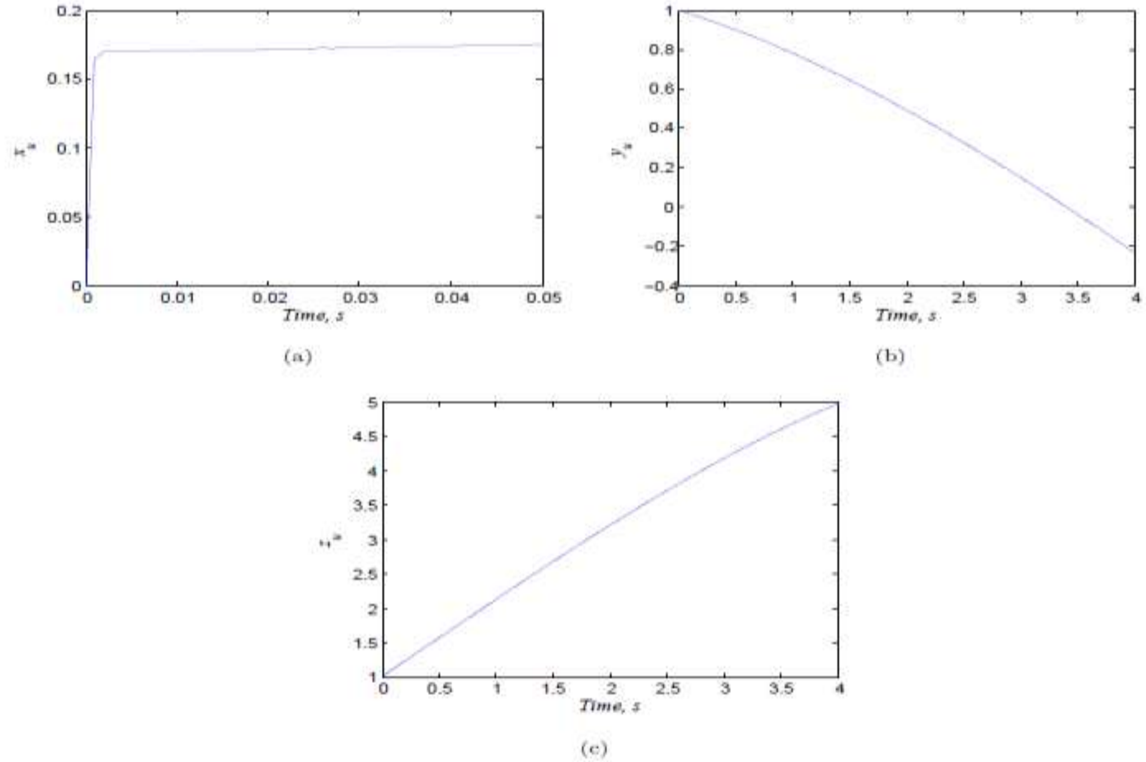

Figure 4: Uniform approximations of x, y and z

Comparison with Actual Solution

To justify the approximations in this project, we also solve the system of ODEs (1) numerically by using the Matlab function *ode15s*. The comparison of the uniform approximations and actual solution can be seen in the Figure 5(a), Figure 5(b) and Figure 5(c).

By using the outer approximation, the reduced ODEs system (6) can be solved in lower computational cost. Through the results shown in the Figure 2(a), Figure 2(b) and Figure 2(c), we can observe that y_o and z_o well-approximate the y and z, but the approximation of x_o is not valid in the boundary layer, that is, $x_o\,(t = 0) \neq x(t = 0)$.

On the other hand, Figure 3 is the inner solutions for the voltages across the capacitors C_1. These solutions satisfy the given initial condition but fail to provide a good approximation after the fast transient period. The matching applied to the outer and inner solutions has provided a better approximation, that is, the uniform approximation. This can be seen in Figure 4(a) and Figure 5(a).

Note here the maximum of absolute error $\hat{e}$, used in the computation is defined as

$$\hat{e} = \max_{0 \leq t \leq T} \{|\mu(t) - \mu_u(t)|\}$$

where $\mu(t)$ represents the solution of full system at time t while $\mu_u(t)$ is the uniform approximation at time t. The maximum of the absolute errors of x, y and z are 0.145, 0.0013 and 0.0096 respectively.

CONCLUSION

In brief, this project is concerned with the application of the singular perturbation theory in reducing the size of dynamic model of the electronic circuits. We have applied the singular perturbation theory to reduce the size of dynamic model of the Chua circuit. The uniform approximation in our study provides a reasonably good approximation with a lower computational cost. The use of the approach discussed in this project can be extended to deal with higher dimensional problem in future, for example, for the VLSI circuits.

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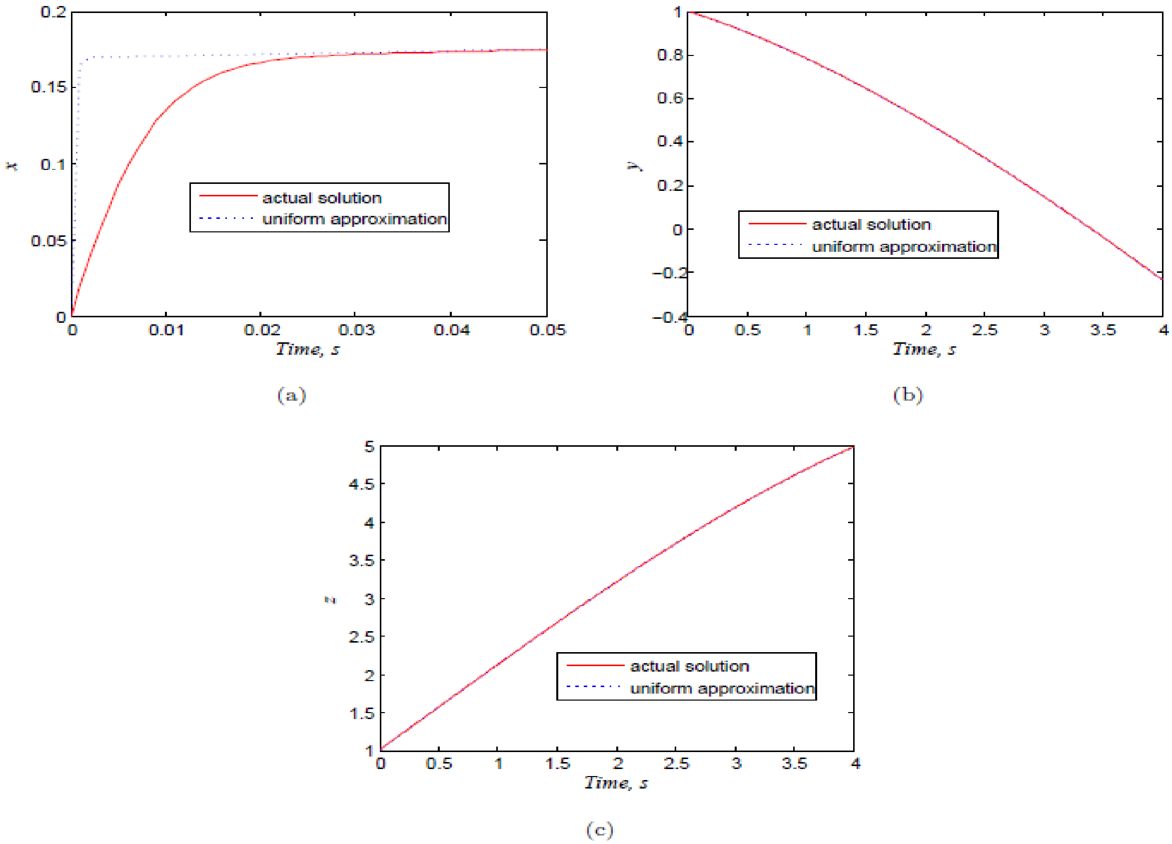

Figure 5: Uniform approximation and numerical solution of system of equations (1) for the voltages across the capacitors C_1, capacitors C_2 and inductor L_1 respectively, with parameters $c_1 = \frac{44}{3}$, $c_2 = \frac{41}{2}, \mu = 2, \beta = 1, a = 0.7, b = 0.24$ and initial conditions: $x(0) = 0$, $y(0) = 1$ and $z(0) = 1$.

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