

# Nonlinear Pushover Analysis of 3D Spatial Frames on Pile Foundations: Integrating Lumped Parameter Soil-Structure Interaction

Pham Nhan Hoa

School of Civil Engineering and Management, International University - VNU HCM, Ho Chi Minh City, Vietnam

Vietnam National University, Ho Chi Minh City, Vietnam

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## ABSTRACT

Assessing the seismic vulnerability of modern structural systems traditionally relies on idealized Fixed Base Building assumptions, which inherently neglect the critical flexibility introduced by the foundation and the surrounding geotechnical environment. This assumption can lead to significant miscalculations in structural demand and capacity. This paper presents an advanced computational framework that integrates a lumped parameter Soil-Structure Interaction model with Nonlinear Static Pushover Analysis to evaluate the true seismic capacity of three-dimensional spatial frames. By representing the multi-layered soil-pile system as a condensed network of dynamic springs and dashpots, the study bridges complex substructure dynamics with performance-based capacity evaluation methodologies. A 9-story steel benchmark building, subjected to varying non-uniform soil conditions, is utilized to demonstrate the proposed analytical framework. The numerical investigation reveals that foundation flexibility significantly alters the fundamental dynamic characteristics of the structural system. In the nonlinear regime, Soil-Structure Interaction significantly shifts the target roof displacement, exacerbates P-Delta effects due to rigid-body base rotation, and redistributes the formation sequence of plastic hinges compared to the conventional Fixed Base Building model. Furthermore, analysis indicates up to a 17.3% increase in column base shear demands under specific soft-soil pile group configurations. The findings provide vital recommendations for structural design engineers, emphasizing that the rigid base assumption may non-conservatively estimate story drift, ultimate ductility, and the overall collapse mechanism of earthquake-resistant high-rise structures.

**Keywords** – Soil-Structure Interaction, Pushover, Dynamic Responses of Pile Foundation

## INTRODUCTION

Earthquake resistance for construction structures is currently a matter of significant concern, given the necessity for resilient infrastructure in seismically active regions globally and regionally. Traditional structural design and analysis methodologies frequently isolate the superstructure from its geotechnical environment to simplify mathematical calculations, operating under the assumption that the column bases are entirely fixed to a rigid foundation. However, to gain a more accurate and realistic understanding of the dynamic response of structures subjected to lateral dynamic loads such as wind or severe earthquakes, both the spatial frame structure above the ground and the pile foundation structure below the surface must be considered simultaneously within a unified computational model. This simultaneous analytical evaluation is recognized as Soil-Structure Interaction, a complex physical phenomenon that fundamentally alters both the inherent dynamic characteristics of the structural system—such as natural periods and mode shapes—and the actual seismic ground motions that are transmitted into the building's superstructure.

Globally, numerous scientific studies and engineering standards have attempted to analyze structures considering Soil-Structure Interaction by employing various numerical methods, primarily including the Winkler foundation method [4], the Direct Method [3] [5] via finite element solid modeling, or by utilizing

condensed lumped parameter models [6]. The Winkler foundation method is conceptually based on the classical theory of beams resting on an elastic foundation, where the surrounding soil medium is represented by a discrete system of uncoupled linear or nonlinear springs. While the primary analytical advantage of this method is that the nonlinearity and heterogeneous behavior of the soil can be simulated without requiring the computational consideration of a massive, semi-infinite continuum problem, it possesses inherent theoretical flaws. Most notably, this method has profound difficulty simulating pile group effects because the uncoupled Winkler model explicitly neglects the cross-interaction and stress-overlap between adjacent piles subjected to lateral and axial loads.

Conversely, the Direct Method models [4] the entire structural and geotechnical problem simultaneously using continuous plane elements for two-dimensional problems or highly complex volumetric solid elements for three-dimensional spatial problems. Because this method explicitly models the soil domain, calculations require a massive amount of computational power and sophisticated resources to solve the vast system of governing differential equations of motion. Consequently, the routine use of this direct computational method is currently quite limited in daily structural engineering design practice, often reserved only for critical infrastructure projects such as nuclear power plants or major dams.

To systematically overcome these computational limitations while preserving geotechnical accuracy, the lumped parameter model represents the infinite soil domain as a discrete macroscopic system consisting of a coupled spring and a dashpot connected directly to the foundation cap. The significant advantage of this specific method is that the complex problem model is highly condensed, rendering it suitable for limited computational resources while still actively capturing the essential kinematic and inertial boundary flexibilities. While previous academic studies have evaluated these dynamic responses using strictly linear time-history analyses in the elastic range, modern performance-based earthquake engineering requires a profound understanding of a structure's behavior deeply into the inelastic and plastic ranges.

This paper directly bridges this critical knowledge gap by integrating Nonlinear Static Pushover Analysis—a core methodology in modern structural dynamics theory—with the three-dimensional spatial frame structures considering the interaction of pile foundations on non-uniform multi-layered soil. This integrated approach aims to provide highly accurate dynamic analysis results that better reflect the actual yielding mechanisms, internal force redistributions, and ultimate collapse states of the structure, serving as a rigorous premise for advanced performance-based seismic design.

## THEORETICAL FRAMEWORK OF SOIL-STRUCTURE INTERACTION

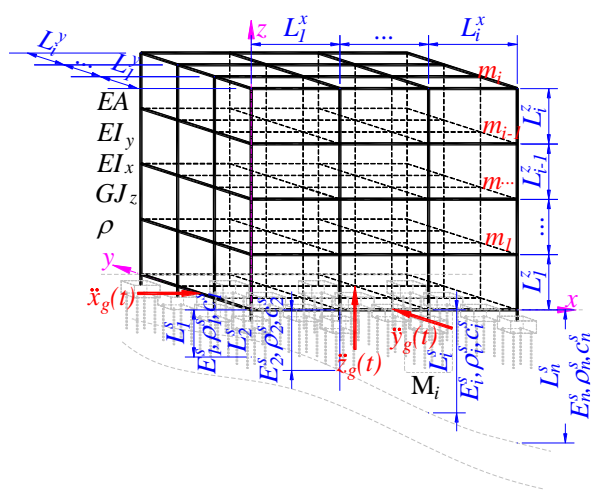


Figure 1: Computational model of 3D frames

The integrated computational model initially defines the structural superstructure as a three-dimensional spatial frame. In the established Cartesian coordinate system, the variables represent the number of structural bays in the orthogonal x and y directions, alongside the total number of elevated stories traversing the z direction. In each structural story, the seismic mass of each floor floor is mathematically transferred to the supporting

girders, and subsequently from these beams to the rigid column nodes, strictly utilizing the established consistent mass matrix rules of the Finite Element Method. The elastic and geometric stiffness of the spatial beam and column elements is mathematically characterized by their specific cross-sectional properties, including axial stiffness parameter, the principal elastic moments of inertia about the local y and x axes, and the Saint-Venant torsional constant governing rotational resistance. Each structural element possesses exactly two terminal nodes, with six independent degrees of freedom at each node—three translations and three rotations—facilitating the formulation of massive corresponding global stiffness and consistent mass matrices for the entire superstructure domain.

For the critical substructure modeling, the structural foundations are located on non-uniform geological layers that vary spatially, which directly and intensely affects the specific dynamic stiffness of the foundation situated under each individual column base. The frequency-dependent dynamic stiffness of each deep pile foundation depends strictly on multiple physical parameters: the exact geometric size of the thick concrete pile cap, the structural length and cross-sectional diameter of the individual piles, and the highly variable geotechnical properties of the surrounding soil continuum, such as the shear modulus, material density, and shear wave velocity. Furthermore, the formulation relies heavily on the specific topological position of the concrete piles within the pile cap arrangement to actively account for the complex pile group interaction effect, where the displacement of one pile influences the stress field of its neighbors. Each discrete foundation is therefore modeled by a complex "dynamic spring" possessing zero physical length, mathematically exhibiting three translational stiffnesses and three rotational stiffnesses about the global coordinate axes, operating entirely alongside corresponding translational and rotational viscous damping coefficients. Compared to the rigid Fixed Base Building model, the total number of dynamic degrees of freedom in the Soil-Structure Interaction analysis strictly increases due to the integration of these six-degree-of-freedom dynamic springs at the structural base.

After rigorously assembling the global mass and global stiffness matrices of the distinct structural elements, the global viscous damping matrix of the spatial structure is determined primarily by utilizing the classical Rayleigh damping method tailored for the beam-column superstructure, which is then mathematically assembled directly with the localized radiation and material damping coefficients of the foundation springs below. The final governing differential equation of dynamic motion for the combined superstructure and foundation substructure system is written in a unified matrix form. This fundamental equation incorporates the combined consistent mass matrix, the assembled global damping matrix, and the total stiffness matrix of both the superstructure and the elastic foundation components, all subjected to the external lumped mass matrices reacting to the directional ground acceleration vectors of the seismic event.

## **THEORETICAL FRAMEWORK OF SOIL-STRUCTURE INTERACTION**

For superstructure formulation, the spatial frame is modeled using standard Finite Element Method (FEM) [5] principles with six degrees of freedom (DOF) per node. Seismic floor masses are distributed to rigid nodes, and element stiffness is derived from cross-sectional properties (axial, bending, and torsional), formulating the global mass and stiffness matrices.

For substructure modeling: Pile foundations are modeled as zero-length, 6-DOF "dynamic springs" and dashpots. Their frequency-dependent dynamic stiffness and damping coefficients directly account for local soil properties (shear modulus, density, wave velocity), pile geometry, and complex pile group interaction effects. Including these springs naturally increases the system's total DOF compared to a Fixed Base Building (FBB) model.

For system assembly, the global mass and stiffness matrices of the frame are combined with the dynamic springs. System damping is determined by coupling the superstructure's Rayleigh damping with the localized viscous damping of the foundations. Finally, these combined matrices are assembled to formulate the unified differential equation of dynamic motion under seismic ground acceleration.

## NONLINEAR STATIC PUSHOVER ANALYSIS INTEGRATION

While the previously described differential equations govern the linear elastic time-history response, modern structural engineering dictates that buildings must safely yield and dissipate energy during maximum considered earthquake events. Nonlinear Static Pushover analysis is a highly effective, performance-based mathematical method specifically designed for evaluating the ultimate structural capacity and ductility in the plastic inelastic range, a methodology heavily featured in contemporary advanced structural dynamics theories. The specific analytical procedure involves subjecting a nonlinear structural model to a monotonically increasing pattern of lateral forces or controlled lateral displacements, continuously mapping the transition from initial elastic behavior through sequential yielding, ultimately leading to full plastic structural collapse or a predefined target displacement.

In standard conventional engineering practice, this critical pushover analysis is executed almost exclusively on a rigid Fixed Base Building. This study fundamentally advances this conventional methodology by substituting those rigid, unyielding boundary conditions with the dynamic, highly compliant flexibility matrices derived from the lumped parameter Soil-Structure Interaction model. The specific spatial distribution of the lateral forces utilized throughout the incremental pushover analysis relies heavily on the fundamental mode shapes and modal participation factors of the entire structure. Because the direct introduction of flexible dynamic springs profoundly alters the structure's global stiffness matrix, the fundamental natural period and the associated structural mode shapes fundamentally and mathematically change. The explicit inclusion of these flexible boundary conditions generally increases the global structural period, directly indicating a much more compliant and flexible structural system compared to the idealized rigid assumption. Consequently, the initial lateral load vector applied during the initial steps of the pushover procedure must be specifically calculated to reflect the unique, evolving fundamental mode of the coupled interaction system, accounting for the rigid-body rotational and translational displacement of the base itself.

To accurately capture the localized inelasticity and material yielding during the rigorous pushover process, mathematically concentrated plastic hinges are systematically assigned to the critical ends of the spatial beam and vertical column elements. These computational hinges follow standardized nonlinear moment-rotation criteria for beams, or complex interacting axial-moment yield surfaces for columns, as meticulously defined in modern advanced structural dynamics and capacity design principles. The crucial incorporation of localized foundation rotation, explicitly arising due to the finite rotational stiffnesses of the foundation's dynamic springs, directly induces additional, severe kinematic lateral displacements at the upper levels of the structure. This drastically heightened inter-story displacement strongly amplifies second-order geometric P-Delta effects, effectively and rapidly accelerating the physical yielding of the critical lower-story columns when directly compared to an idealized, purely fixed condition.

## COMPUTATIONAL MODEL AND NUMERICAL INVESTIGATION

To rigorously quantify these theoretical concepts, the structure investigated within this study is a comprehensive 9-story spatial benchmark building constructed entirely from structural steel. The complex spatial geometry features exactly 5 distinct bays traversing the global x direction, and 5 bays traversing the global y direction, with each individual structural bay spanning a uniform length of exactly 9.15, generating a total structural footprint width of 45.73m. The first story features a height of 4.49m, while the subsequent upper stories maintain a constant height of 3.96m, culminating in a total building height of 37.17m above the foundation cap. The total number of unrestrained dynamic degrees of freedom for the Fixed Base Building analytical model is exactly 2160; however, the necessary inclusion of the foundational dynamic springs in the interaction model heavily increases this matrix size to 2376 operational degrees of freedom. The primary structural wide-flange members are modeled with an elastic modulus of 200GPa, incorporating an inherent structural damping ratio of two percent. For the critical substructure elements, the supporting frame actively utilizes reinforced concrete pile foundations conforming to grade B27.5, possessing a specific pile elastic modulus of 35MPa, a uniform physical pile diameter of 0.4m, and an assumed density of the thick pile cap set at  $2500\text{kg/m}^3$ .

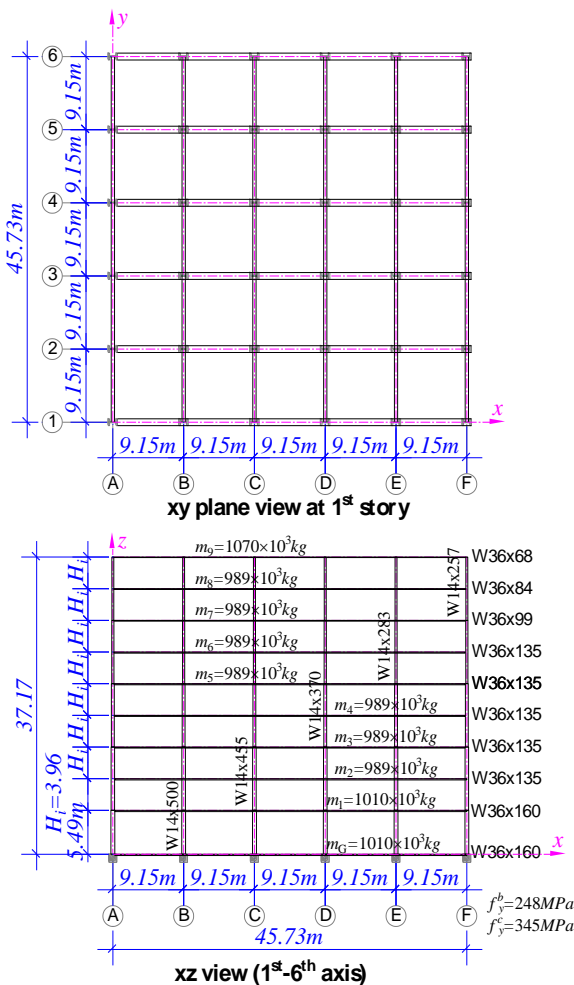


Figure 2: Computational finite element model of the 9-story 3D steel spatial frame, indicating the grid of lumped parameter dynamic springs supporting the base columns.

To provide a robust, comprehensive parametric assessment of the geotechnical variables, four entirely separate and distinct computational cases are mathematically analyzed to cover a wide spectrum of physical geotechnical environments, ranging specifically from very soft alluvial soils to highly stiff strata. Soil Type 1 fundamentally represents extremely soft conditions, exhibiting a low intrinsic shear modulus of 10.71MPa alongside a sluggish wave velocity of only 74m/s. In stark contrast, Soil Type 4 represents highly stiff, competent conditions with a significantly higher shear modulus of 37.50MPa and rapid shear wave velocities reaching 139m/s. Various physical pile cap configurations are strictly evaluated across these diverse soil profiles. For example, a 3 by 3 pile group arrangement featuring a center-to-center spacing of 2.0m and a deep pile length of 17.55m yields a calculated horizontal stiffness of 1446.3kN/m, serving as the essential, quantifiable boundary condition for the foundation springs utilized in the subsequent pushover analysis. The first fundamental natural period of the pristine structure in the rigid Fixed Base Building model is documented at exactly 1.735s, a baseline metric that directly shifts as foundation flexibility is introduced.

## RESULTS AND DISCUSSION

By directly advancing the linear time-history analysis into a comprehensive nonlinear pushover framework, it becomes immediately evident that base foundation flexibility strictly dictates the ultimate energy capacity, target displacement, and sequential failure mechanism of the spatial frame. Linear responses extracted from the time-history dynamic data initially show that extreme differences between the fixed base and flexible base systems are heavily concentrated specifically within the ultimate shear force and the resultant column base moment responses acting in the primary working plane of the spatial frame structure.

For explicit numerical instance, under standard linear dynamic conditions, the recorded first-story column base shear force for the idealized Fixed Base model is measured at 450.3kN. However, when evaluating the

identical structure utilizing the interacting soft soil model with a 2 by 2 extended pile arrangement featuring a length of 29.25m and a spacing of 3 times the pile diameter, the identical column base shear radically spikes to 528.2kN. This represents a massive, potentially catastrophic increase of 17.30% strictly due to the interactive boundary conditions.



Figure 3: Representation of the base shear force comparison across multiple foundation configurations, demonstrating the 17.30 percent increase in demand for soft soil environments

When projecting this 17.30% internal force increase directly into the nonlinear pushover regime, the structural implications are profound. This artificially elevated internal force demand dictates that localized plastic hinging at the critical column bases must occur at substantially lower global base shear loading levels than the Fixed Base models mathematically predict. Because the foundation nodes are yielding to rotational demands much earlier, the structural system's total overstrength factor is severely reduced, compromising the frame's theoretical margin of safety against total dynamic collapse. The yielding sequence, rather than being uniformly distributed across the lower third of the structure, violently concentrates at the base due to the amplified lateral drift and resulting massive P-Delta moments.

Furthermore, the performance-based structural target displacement, which essentially represents the mathematical maximum expected roof displacement under a highly specific design earthquake scenario, is definitively proven to be highly sensitive to the underlying foundational model. Baseline linear dynamic analyses explicitly indicate that maximum absolute top displacements increase from a baseline of 4.291cm in the Fixed Base model to an elevated 4.408cm in the highly flexible structural configurations. In the extended nonlinear pushover analysis, this specific kinematic disparity magnifies significantly. The physical rigid-body rotational motion of the entire flexible foundation cap mathematically contributes heavily to the total roof drift. This means a vast, disproportionate portion of the calculated ultimate target displacement is derived artificially from harmless foundation compliance rather than the actual, energy-dissipating plastic structural deformation within the steel beams and columns. Consequently, assessing true structural ductility strictly based on total absolute roof displacement within an interaction model will highly and dangerously overestimate the structural frame's actual physical ability to successfully dissipate destructive seismic energy.

## CONCLUSION

The highly advanced computational finite element model meticulously presented in this comprehensive study provides a vastly superior and robust mathematical framework for accurately analyzing the true dynamic inelastic response of a three-dimensional spatial frame structure by explicitly considering soil-structure interaction through the implementation of frequency-dependent dynamic springs. Depending strictly on the geometric shape of the foundation cap, the arrangement of the piles, and the diverse dynamic characteristics of the specific multi-layered soil beneath the foundation, the calculated stiffness values of these discrete dynamic springs vary significantly from very low to exceptionally high. This massive variation fundamentally and permanently alters both the natural elastic vibration periods and the global dynamic response kinematics of the

spatial structure when directly compared to simplified traditional models that completely fail to consider foundational interaction.

By successfully integrating this condensed lumped parameter interaction model with Nonlinear Static Pushover Analysis, this research reveals profound, hidden differences in the expected yielding patterns and ultimate plastic collapse mechanisms of massive spatial frames. When properly designing multi-story high-rise buildings for severe earthquake resistance, the highly substantial numerical errors explicitly documented in base shear—reaching up to 17.3%—and the similarly distorted column base moments observed in traditional fixed-base computational models strongly dictate a necessary paradigm shift. It strongly suggests that professional design engineers must explicitly and mathematically account for the highly detrimental additional internal forces strictly generated by the localized subsoil interaction effects on the elastic superstructure. Adopting this highly integrated pushover-interaction methodology is not merely an academic exercise, but rather a vital imperative for generating safe, accurate, and truly reliable performance-based engineering designs, particularly for vulnerable spatial structures located in highly active seismic regions uniquely characterized by dangerously deep, highly flexible soft soil deposits.

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