Exploring Mathematical Epistemology of Grade 9 Students in Validating the Pythagorean Theorem

[1]Sittie Nashebah B. Guinto., [2]Bartolome L. Cagas

[1]Bacolod-Kalawi National High School, Bacolod-Kalawi, Lanao del Sur, Philippines

[2]Institute of Science Education, Mindanao State University, Marawi City, Philippines

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ABSTRACT

Understanding how students justify, evaluate, and validate mathematical knowledge is central to effective mathematics instruction. This study examined the mathematical epistemology of Grade 9 students from a public national high school in Lanao del Sur, Philippines, as they validated the Pythagorean Theorem through a guided investigation. The task design intentionally utilized eight distinct side-length parity sets, including non-right triangle distractors to test the exclusionary logic of the theorem. Students' written reflections were analyzed and categorized into two epistemic stances: practical epistemic reasoning and formal epistemic reasoning. The findings indicate that students' mathematical epistemology is highly context-dependent and shaped by task design. While practical reasoning focused on visual representation and empirical evidence was initially dominant in initial tasks, a significant epistemic transition toward formal reasoning (theoretical justification and pattern recognition) occurred in tasks requiring the verification of universal validity. However, the study also reveals that students are prone to developing heuristic rules based on numerical patterns, such as parity (even/odd), rather than relying solely on formal mathematical properties. These results highlight the critical role of structured inquiry and the need for rigorous verification tasks that account for potential instructional-induced biases. The findings offer implications for mathematics instruction, emphasizing the importance of bridging practical engagement and formal reasoning to foster a more coherent and robust mathematical epistemological awareness.

Keywords: Mathematical Epistemology, Epistemic Cognition, Theory-Evidence Coordination, Model-Based Reasoning, Inquiry-Based Learning, Mathematics Education

INTRODUCTION

Understanding the student's mathematical epistemologies in validating mathematics principles is drawing attention among mathematics teachers, educators and researchers for it is essential in promoting deep conceptual learning. Epistemology is a branch of philosophy that explores the nature, origin, and justification of knowledge (Lemos, 2007). Within the mathematical context, epistemology examines how mathematical knowledge is acquired, justified, and what constitutes its truth or validity (Hersh, 1997). In a practical sense, it addresses how mathematical principle was formulated, evaluated, and validated through trial and error exploration looking for patterns and use reasoning for validation. In the classrooms, students often engage in processes that reflect different epistemic stances ranging from practical (classroom learning-based experiences) to formal (deductive derivation as practice by professional mathematician). In this study we adopted Baker's (2022) definition of stances in the mathematical context as structured sets of assumptions that guide how we treat, interpret, and justify mathematical knowledge in both pure and applied setting. Practical mathematical epistemology focuses on how mathematical knowledge is generated, used, and justified in real-world contexts moving beyond abstract philosophical debates to examine mathematics as it is practiced by mathematicians, educators, and practitioners in fields like science, engineering, and industry (Sierpinska & Lerman, 1996). Formal mathematical epistemology examine how knowledge is constructed through symbolic systems and logical proof, focusing on the foundational structures that underpin mathematical validity (Robert, 2023).

The present study focused on mathematical epistemologies demonstrated by students in validating Pythagorean theorem that will serve as a framework of understanding how Grade 9 students utilized mathematics learning experiences, reasoning, and their understanding of Pythagorean theorem. The epistemic stances are examined through Grade 9 student's attempt to validate Pythagorean theorem through parity of the length of the sides of the triangle using trial and error exploration. The verificatory task used eight sets of right triangle side lengths varying in parity (even-even, even-odd, odd-odd combinations), requiring students to test the theorem and document observations of patterns. This task constituted the experiential basis for students' subsequent reasoning.

In the context of geometry, one of the most foundational and frequently applied theorems is the Pythagorean Theorem, which states that in a right triangle, the square of the hypotenuse equals the sum of the squares of the two legs. This theorem is not only a mathematical statement but also a conceptual bridge between abstract reasoning and real-world application, serving as an ideal framework to examine students' epistemological processes.

Despite Pythagorean Theorem's familiarity among Grade 9 students as it is explicitly taught in mathematics (i.e. Geometry), many students struggle to comprehend and validate it meaningfully. They can often recite the formula ($a^2 + b^2 = c^2$) but fail to grasp its logical foundation or its practical applications. Specifically, the students struggle to perform algebraic manipulation of the Pythagorean relation to find what is asked or unknown among the three sides of a right triangle. Study showed that 60.8% of the students committed strategy errors indicating a great difficulty in manipulating the Pythagorean relation, this was followed by conceptual errors at 53.3% and calculation errors at 51.7%, reflecting challenges in understanding concepts and performing operations accurately (Taamneh & Díez Palomar, 2024).

Despite familiarity with the Pythagorean Theorem, many students struggle to comprehend and validate it expressively, often treating it as a memorized rule rather than a derived relationship (Taamneh & Díez Palomar, 2024; Fadillah & Rifki, 2024). This is further complicated by (Hutapea, Suryadi, & Nurlaelah, 2015), errors in problem-solving (Taamneh & Díez Palomar, 2024), and gaps in conceptual understanding (Sandra, 2025). These challenges underscore the need to explore not just what students know, but how they come to know and justify mathematical truths.

Existing literature highlights these difficulties; however, few studies examine the epistemological dimension of students' understanding. A systematic review by Bariyah & Prabawanto (2024) reinforces the prevalence of these obstacles. This research aims to address this gap by exploring Grade 9 students' epistemologies in validating the Pythagorean Theorem, specifically how they use practical and formal reasoning, thereby contributing a more nuanced understanding of their cognitive and philosophical engagement with mathematical knowledge.

While existing literature acknowledges students' difficulties with the Pythagorean Theorem, significant deficiencies remain in how these challenges are connected to students' epistemologies, particularly within the context of Grade 9 public high school students. The current body of research, while valuable, primarily concentrates on identifying common misconceptions, procedural errors, and conceptual gaps (Taamneh & Díez Palomar, 2024; Fadillah & Rifki, 2024; Hutapea, Suryadi, & Nurlaelah, 2015; Sandra, 2025). These studies effectively highlight what students struggle with, but offer limited insight into how students construct, validate, and justify mathematical knowledge related to the theorem.

To address these deficiencies, the present study aims to provide an in-depth exploration of Grade 9 public high school students' epistemologies in validating the Pythagorean Theorem. By examining how students use practical and formal reasoning to justify the theorem, the types of evidence they find convincing, and how these approaches manifest in their problem-solving, this research contributes a more nuanced understanding of students' cognitive and philosophical engagement with mathematical knowledge.

This study will benefit mathematics educators, curriculum developers, and educational researchers interested in epistemology, cognition, and mathematics instruction. By identifying how students' reason and validate mathematical truths, teachers can design lessons that integrate both experiential and formal reasoning. The findings will also help curriculum designers develop materials that balance conceptual understanding and procedural fluency. Ultimately, this study aims to enhance teaching practices that cultivate mathematically literate and epistemologically aware learners.

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Research Objectives

The purpose of the study is to investigate the students' mathematical epistemology. Specifically, the study aims to explore and categorize "epistemic stances" of Grade 9 students from Bacolod-Kalawi National High School, Bacolod-Kalawi, Lanao del Sur, Philippines when validating the Pythagorean theorem.

Theoretical Framework

This study is grounded on mathematical epistemology offering a comprehensive lens to examine and understand students' epistemic stances during validation of Pythagorean theorem.

Mathematical epistemology refers to how mathematical knowledge is acquired, justified, and what constitutes its truth or validity (Hersh, 1997). Within the mathematical epistemology framework, we specifically include Epistemological Beliefs theory by Hofer & Pintrich (1997) and Schommer (1990). The theory explores how individuals perceive the nature and acquisition of knowledge whether as fixed or evolving, simple or complex, certain or uncertain, and whether it comes from authority or is self-constructed.

In mathematics education, these beliefs shape how students justify and validate truths; for instance, some may view the Pythagorean Theorem as an unchanging rule from authorities, while others recognize it as a concept that can be proven through logic and experience.

This study uses the theories cited to classify students' epistemic stances in validating Pythagorean theorem through trial and error exploration involving parity of the measurements of the length of the right triangle sides. These classifications help interpret how learners justify their understanding of the Pythagorean Theorem.

Conceptual Framework

Figure 1 presents the conceptual framework guiding this study of Grade 9 students' epistemic stances during the validation of the Pythagorean Theorem. The framework illustrates specifically how mathematical task in validating Pythagorean theorem shapes students' epistemic engagement and reasoning that leads to observable reasoning pattern outcomes.

At the center of the framework is the verification of the Pythagorean Theorem of the form ($a^2 + b^2 = c^2$), which assessed students validations skills, mathematical knowledge and understanding of mathematics epistemologies. This validation task framing activates students' epistemic resources such as prior conceptual knowledge and epistemological stances which are needed in validating mathematical principles.

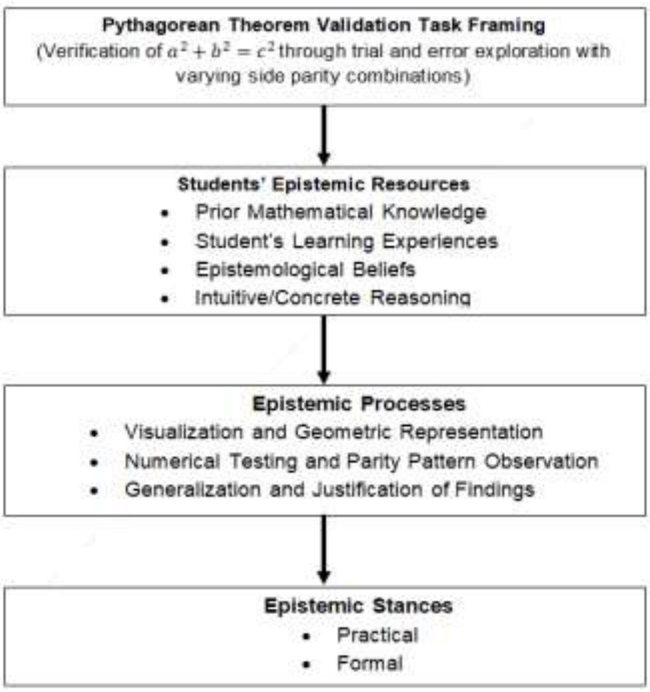

Figure 1. Conceptual Framework

These epistemic resources and epistemological stances enable students to engage in justifying and validating mathematical principles which may resulted to distinct epistemic reasoning patterns, either formal epistemic reasoning characterized by the use of mathematical verification and validation by example or practical epistemic reasoning characterized by students' own epistemologies other than the standard mathematical epistemologies used by experts.

Overall, the framework highlights how students justify and validate mathematical principles through the use of their epistemic resources and engagement in epistemic processes, enabling analysis of how students justify and validate mathematical principles like Pythagorean theorem.

METHODOLOGY

Research Design

This study employed an explorative descriptive research approach utilizing qualitative data in the form of responses to the open-ended questions frame to explore Grade 9 students' mathematical epistemology at every stages of the validation process of Pythagorean theorem. This approach allowed analysis of not only whether students could apply the theorem $a^2 + b^2 = c^2$ but also how they use empirical evidence and logical reasoning to support their understanding, aligned with constructivist and epistemological perspectives.

Participants

The study was conducted during the fourth quarter of the 2024–2025 academic year at Bacolod-Kalawi National High School, located in Bacolod-Kalawi, Lanao del Sur, Philippines. The school is recognized for its engagement in academic and extracurricular activities at division and regional levels. A total of 30 Grade 9 students (14 males and 16 females; mean age = 15 years) participated in the study. Due to progressive spiraling curriculum of public schools, basic foundational knowledge on algebra, statistics and probability and geometry were introduced in grade 7 and 8. The Pythagorean theorem was introduced in grade 9 in the second quarter providing them with background knowledge and understanding and problem-solving skills. This advanced background knowledge and understanding was considered during data interpretation, as prior learning experiences may influence epistemic reasoning.

Data Gathering

Data collection was embedded within a classroom-based investigation designed to capture students' spontaneous reasoning. Before the start of the investigation, the participants were provided with a brief refresher on the Pythagorean Theorem ($a^2 + b^2 = c^2$) and the general concept of mathematical verification. This ensured a baseline understanding of the theorem's formula. However, no formal lecture or explicit instruction on mathematical epistemological approaches such as how to specifically structure their justifications or validate their claims was provided prior to the activity. This was intended to capture the students' existing epistemic resources and authentic ways of validating mathematical knowledge.

The investigation began with the students completing a verification task independently. They worked through eight sets of side lengths with distinct parity combinations: (1) even-even-even, (2) even-even-even, (3) odd-odd-even, (4) odd-odd-even, (5) odd-even-odd, (6) odd-even-odd, (7) even-odd-odd, and (8) even-odd-odd. To ensure a rigorous validation process, the research instrument deliberately integrated non-right triangle sets as discriminatory variables. This was designed to test the ability to distinguish between valid Pythagorean triples and mathematical distractors thereby verifying the exclusionary function of the relation $c^2 = a^2 + b^2$ rather than merely testing rote calculation. For each set, they calculated a^2, b^2, and c^2 checked if $c^2 = a^2 + b^2$, and noted the parity of each side to identify emerging patterns. To capture students' existing epistemic resources, no formal lecture or explicit instruction on mathematics epistemological approaches in validating the theorem was provided prior to the activity.

Following the verification task, students proceeded to a five-item open-ended questionnaire designed to explore their reasoning about the purpose of (1) visualizing triangles, (2) testing specific examples, (3) verifying the theorem, (4) analyzing parity patterns, and (5) constructing generalizations. All written responses were completed individually to ensure they reflected students' personal epistemological perspectives. Throughout the activity, field notes were recorded to document observable behaviors such as problem-solving strategies, instances of confusion or insight, and time allocation across task sections.

Upon completion, all instruments were collected and coded with unique identifiers to protect participant anonymity. Responses were then compiled and prepared for transcription, with handwritten entries transcribed verbatim to preserve the authenticity of students' reasoning. The (SMEVPT) was developed by the researchers and reviewed by experts in mathematics education to ensure content relevance, clarity, and alignment with the study's theoretical and conceptual frameworks.

Data Analysis

Students' written responses were transcribed verbatim and analyzed using thematic analysis following Braun and Clarke's (2006) six-phase framework. Initial coding focused on identifying recurring themes related to students' justifications, use of evidence, and interpretations of experimental outcomes. These themes were then examined in relation to epistemic criteria drawn from the study's theoretical framework.

Responses were classified into two epistemic stances: Formal epistemic reasoning and practical epistemic reasoning. Formal epistemic reasoning characterized by the use of mathematical verification and validation by example. On the other hand, practical epistemic reasoning characterized by students' own epistemologies other than the standard mathematical epistemologies used by experts. These classifications were informed by established frameworks in mathematical epistemology and epistemic cognition (Lising & Elby, 2005; Sandoval, 2004).

To enhance trustworthiness, the data were independently coded by two researchers. To establish the reliability of the qualitative coding and thematic analysis of the students' epistemic stances, the researchers employed inter-rater reliability measures. Rather than simply describing the percentage of agreement, Cohen's Kappa (κ) was calculated to account for any agreement occurring by chance. The analysis yielded a Kappa coefficient of 0.85, which indicates an almost perfect agreement between the raters. Three experts in mathematics education subsequently reviewed the coding scheme and category assignments to ensure conceptual clarity and alignment with the study's epistemic framework. Discrepancies were discussed and resolved through consensus, ensuring consistency and credibility of the analysis (Nowell et al., 2017). Frequencies and percentages of each reasoning pattern were calculated to provide a descriptive overview of students' epistemic engagement across the five questions.

Ethical Considerations

As per international standards and institutional guidelines, ethical approval for this study was obtained prior to data collection. Informed consent was secured from students' parents or legal guardians, and assent was obtained from the participating students themselves.

All participants were assured of confidentiality and anonymity, with data stored and processed in accordance with privacy regulations. Participation was voluntary, and students were informed they could withdraw from the study at any time without penalty or impact on their academic standing. All data collected has been organized and preserved by the researchers for potential future analysis, while maintaining strict safeguards to protect participant identities.

RESULTS AND DISCUSSION

Results

Students' epistemic stances in validating the Pythagorean Theorem was analyzed using their responses to five open-ended questions and a generalization task (total participants = 30). Responses were grouped thematically and classified as practical epistemic reasoning (characterized by visualization, real-world application focus,

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and procedural accuracy) or formal epistemic reasoning (marked by theory-evidence coordination, pattern recognition, and justification of mathematical principles). A subset of responses reflecting inconsistent or isolated reasoning was noted where applicable. Table 1 presents the distribution of themes, reasoning patterns, frequencies, and representative student responses.

Table 1. Students' epistemic reasoning patterns in validating the Pythagorean theorem

Question	Themes of Students' Answers	Epistemology	Number (%) of Students	Example of Student's Answer
1. Why do we need to draw/illustrate the right triangle with assign values for side's length?	Visualization and Understanding	Practical Epistemology	9 (30%)	"Drawing a right triangle with assign values for side's length helps visualize the relationships of the sides. It makes it easier to understand the concepts of Pythagorean theorem." (Respondent 14)
	Knowing what is asked in the problem and solve it using the Pythagorean Formula	Practical Epistemology	8 (26.67%)	"By assigning values to the side's length of a right triangle, it allows me to apply the Pythagorean theorem formula to identify and solve for unknown sides in the problem." (Respondent 24)
	Avoiding Errors for Accuracy	Practical Epistemology	8 (26.67%)	"Sa pamamagitan ng pag sulat ko sa values ng mga sides ng right triangle, natutulungan niya akong maka focus sa specific unknown sa problem na sino-solve ko to avoid errors and confusion. I can focus on the problem and solve accurately." (Respondent 1)
	Communication and Explanation	Practical Epistemology	5 (16.67%)	"A visual representation with assigned values in the side's length of a right triangle makes it easier to communicate mathematical problems and solutions to others." (Respondent 10)
2. Why do we need to make use of specific examples of assign values of the sides' length of the right triangle to test the relation $c^2 = a^2 + b^2$ (Pythagorean theorem)?	Verification and Proving	Formal/ Standard Epistemology	8 (26.67%)	"Using specific examples allows us na e verify ang accuracy ng Pythagorean theorem's by directly substituting the values of the side's length in the Pythagorean theorem and checking if the equation holds true." (Respondent 26)
	Understanding the Pythagorean Relation for Application to real-	Practical Epistemology	9 (30%)	"Applying the theorem to specific examples helps us understand kung paano e so- solve ang practical or real-world problems

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	world problems			na may right triangles. Nako-connect nito ang gap between the concept and its real-world applications." (Respondent 21)
	Intuition and Building Confidence	Practical Epistemology	7 (23.33%)	"Dahil nakaka build ito ng confidence para ma apply ko ang pythagorean theorem ng tama. Dahil when I am working through examples it strengthens my understanding of the relationships between the sides. Successful problem-solving raises a better intuitive grasp of the theorem." (Respondent 18)
	Identifying Consistency of Patterns of Relationships	Formal/ Standard Epistemology	3 (10 %)	"To see patterns in the relationship between sides. Nakaka tulong ang examples para ma recognize natin how changes in one side affect the others." (Respondent 1)
	Solving problems correctly and accurately	Practical Epistemology	3 (10 %)	"Specific examples provide practice in applying the theorem para mahanap yung missing values ng side's length at ma improve ang problem-solving skills to have an accurate calculation." (Respondent 10)
3. Why do we need to test/verify the mathematical relation $c^2 = a^2 + b^2$ (Pythagorean theorem)?	Verification and Confirmation	Formal/ Standard Epistemology	8 (26.67%)	"Testing the Pythagorean theorem verifies its accuracy and ensures na nagho- holds ito sa lahat ng right triangles, confirming its reliability or validity as a fundamental mathematical principle." (Respondent 14)
	Building Understanding and Confidence	Formal/ Standard Epistemology	6 (20%)	"Verifying the theorem through testing builds a deeper understanding of its underlying concepts at nakaka increases din ito ng confidence sa pag apply natin ng pythagorean theorem sa pag solve ng problems." (Respondent 26)
	Identifying Consistency of Relationships Patterns	Formal/ Standard Epistemology	5 (16.67%)	"Kapag tini-test kasi ang theorem nari-reveal yung relationships ng sides ng right- triangle at naha-highlights yung consistent pattern na dini described ng theorem." (Respondent 28)

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	Understanding the Application in Real-world Problems	Practical Epistemology	8 (26.66%)	"By testing the Pythagorean theorem nakikita natin yung practical application nito sa pag solve ng real-world problems involving distances, lengths, and angles." (Respondent 18)
	Develop Problem-Solving Skills and Improve Mathematical Abilities	Formal/ Standard Epistemology	1 (3.33%)	"Testing the theorem helps us develop problem-solving skills and improve our mathematical abilities."
4. Why do we need to take a look/notice the testing/verification results and pattern/trend of parity (even, odd) of the sides' length of the right triangle?	Discover Patterns and Relationships	Formal/ Standard Epistemology	9 (30%)	"Examining the parity of right triangle sides reveals patterns and relationships between the evenness or oddness of side lengths and the theorem's outcome." (Respondent 20)
	Understand the Theorem.	Formal/ Standard Epistemology	6 (20%)	"Observing parity trends in the testing results can enhance our understanding of the Pythagorean theorem." Respondent 21)
	Checking the Theorem and Identifying Limitations	Formal/ Standard Epistemology	6 (20%)	"By carefully examining the parity of side lengths and their relationship to the theorem's results, through this pwede natin ma-a identify yung limitations or exceptions ng Pythagorean theorem." (Respondent 10)
	Checking the Theorem and Identifying Limitations	Formal/ Standard Epistemology	5 (16.67%)	"Kasi nade-develop yung critical thinking and problem-solving skills natin kapag nag a-analyze tayo ng parity of side lengths in right triangles. It teaches us to look for patterns, analyze data, and draw conclusions from observations." (Respondent 28)
5. Construction of generalization/conclusion. Base from the testing/verification results and pattern/trend of parity (even, odd) of the sides' length of the right triangle,	**A. Pair 1 & 2 (Both Legs are even):**			
	Pythagorean Theorem doesn't work with even legs (Pair 1 & 2: Both Legs are even)	Formal/ Standard Epistemology	19 (63.33%)	"I tried a few examples with both legs of a right triangle being even numbers, and it just didn't work out. The Pythagorean Theorem seems like it doesn't work with even legs." (Respondent 5)
	The sum of the squares of the two legs is shorter than the square of the	Formal/ Standard Epistemology	6 (20%)	"When I square the even legs and add them together, the answer is always smaller than the square of the hypotenuse. The theorem isn't

construct/write one sentence only generalizing the testing/verification results and pattern/trend of parity (even, odd) of the sides' length of the right triangle for the pair numbers 1 and 2, 3 and 4, 5 and 6, and 7 and 8. Write them in enumerated form.	hypotenuse (Pair 1 & 2: Both Legs are even).			working correctly when the two legs of a right triangle are both even." (Respondent 16)
	Pythagorean Theorem Works with different parity (Pair 1 & 2: Both Legs are even).	Formal/ Standard Epistemology	5 (20%)	"It seems to work differently when both legs are even, the legs must be with different parity for the theorem to work." (Respondent 10)
	B. Pair 5 & 6 (Both Legs are odd):			
	Pythagorean Theorem doesn't work with odd legs (Pair 5 & 6: Both Legs Odd).	Formal/ Standard Epistemology	19 (63.33%)	"I've tried a few triangles with both legs being odd, but it doesn't seem to work. The Pythagorean Theorem is supposed to be true for all right triangles, but it's not working for these ones. Maybe the theorem only works for triangles with even legs." (Respondent 16)
	The sum of the squares of the two legs is longer than the square of the hypotenuse (Pair 5 & 6: Both Legs Odd).	Formal/ Standard Epistemology	6 (20%)	"The sum of the squares of the legs is always bigger than the square of the hypotenuse. It seems like odd numbers does not work with Pythagorean Theorem." (Respondent 30)
	Pythagorean Theorem Works on Different Parity of the Legs length	Formal/ Standard Epistemology	5 (20%)	"It seems like the Pythagorean Theorem has a special rule about parity. It doesn't seem to work the same way when both legs are odd or both legs are even. It is only true when the parity of two legs are opposite." (Respondent 10)
	C. **Pair 3 & 4 (One Leg Even, One Leg Odd):** D. **Pair 7 & 8 (One Leg Odd, One Leg Even)**			
	Pythagorean Theorem Works on Different Parity of the Legs length	Formal/ Standard Epistemology	27 (90%)	"The Pythagorean Theorem works best when you have one even leg and one odd leg." (Respondent 2)
			3 (10 %)	No answer

For the first question, "Why do we need to draw/illustrate the right triangle with assigned values for side's length?", all responses were categorized as practical epistemic reasoning. Four themes emerged: 9 students (30%) focused on visualization and conceptual understanding (e.g., " Drawing a right triangle with assign values for side's length helps visualize the relationships of the sides. It makes it easier to understand the concepts of Pythagorean theorem" - Respondent 14); 8 students (26.67%) emphasized using diagrams to identify unknowns and apply the formula; another 8 students (26.67%) noted illustrations reduce errors and improve accuracy; and 5 students (16.67%) highlighted their role in communicating solutions.

Regarding the second question, "Why do we need to make use of specific examples to test the relation $c^2 = a^2 + b^2$?", 19 students (63.33%) demonstrated practical reasoning with 30% linking examples to real-world application, 23.33% focusing on building confidence and intuition, and 10% emphasizing skill improvement. The remaining 11 students (36.67%) showed formal reasoning: 26.67% referenced verifying the theorem's accuracy through substitution (e.g., "Using examples allows us to verify the accuracy of the theorem" - Respondent 26), while 10% focused on identifying consistent patterns between sides.

For the third question, "Why do we need to test/verify the Pythagorean theorem?", 20 students (66.67%) displayed formal reasoning. Of these, 26.67% emphasized confirming the theorem's universal validity (e.g., "Testing ensures it holds for all right triangles" - Respondent 14), 20% noted it builds conceptual understanding and confidence, 16.67% focused on revealing consistent relationships, and 3.33% linked verification to developing mathematical abilities. The remaining 8 students (26.66%) demonstrated practical reasoning, with all highlighting application to real-world problems involving distances and lengths.

In response to the fourth question, "Why do we need to notice parity patterns of the sides' length?", all 30 responses were classified as formal/standard epistemic reasoning. Nine students (30%) focused on discovering patterns and relationships (e.g., "Examining parity reveals connections between side evenness/oddness and outcomes" - Respondent 20); 6 students (20%) noted it enhances overall understanding of the theorem; another 6 students (20%) emphasized identifying potential limitations; and 5 students (16.67%) highlighted development of critical thinking skills.

For the generalization task, which required one-sentence conclusions for four side parity pairs, all responses were categorized as formal/standard reasoning, though notable inconsistencies emerged. Students correctly verified that the theorem holds true for all valid Pythagorean triples (Items 1, 2, 3, 5, 6, 7, & 8). However, when presented with item 4 (3, 5, 7), which is mathematically invalid, the majority correctly observed that the relation does not hold. This indicates that students base their generalizations strictly on the results of their calculations.

Overall, students' approaches across the investigated scenarios were predominantly classified as formal mathematical reasoning, with higher frequencies observed in tasks related to the lengths of the legs, the length of the hypotenuse, comparison of calculated and measured values, and geometric construction accuracy. Approaches categorized as practical epistemic reasoning occurred more frequently in tasks concerning abstract properties of triangles and in general or procedural explanations.

DISCUSSIONS

Why do we Need to Draw/Illustrate the Right Triangle with Assigned Values for Side’s Length?

Students’ uniform focus on practical epistemic reasoning for this question indicates a well-developed orientation toward concrete, visual understanding of mathematical concepts. The majority framed diagrams as tools to clarify geometric relationships, apply the formula to unknowns, reduce errors, and communicate solutions reflecting a constructivist approach where knowledge is built through perceptual and procedural engagement (Piaget, 1972).

From an epistemological perspective, these responses align with practical epistemology beliefs, where students prioritize tangible connections between abstract formulas and observable representations. This pattern is consistent with prior research noting that visual aids help learners assimilate new information into existing cognitive schemas (Hutapea et al., 2015). However, the absence of formal reasoning themes here suggests that while students recognize the utility of illustrations, few explicitly link them to verifying the theorem’s logical or universal foundations highlighting a gap between practical application and conceptual depth.

These findings underscore the importance of framing visualization not just as a procedural tool, but as a bridge to formal reasoning. Instructional practices that guide students to connect diagrams to mathematical principles (e.g., explaining why right angles are necessary for the theorem to hold) can strengthen their epistemological awareness of how visual representations support theoretical validation (Creswell & Poth, 2018).

Why do we Need to Make Use of Specific Examples to Test the Pythagorean Relation $c^2 = a^2 + b^2$?

The split between practical and formal reasoning in responses to this question reflects a range of epistemic engagement. Students emphasizing practical reasoning focused on real-world application, confidence-building, and skill improvement consistent with social constructivist views that link learning to contextual use (Vygotsky, 1978). Those demonstrating formal reasoning framed examples as tools for verification and pattern identification, aligning with Epistemological Beliefs Theory (Hofer & Pintrich, 1997) where knowledge is viewed as provable and systematic.

Similar to findings by Taamneh & Díez Palomar (2024), many students initially prioritize procedural mastery over conceptual justification. However, the subset recognizing pattern consistency suggests emerging ability to coordinate empirical evidence with theoretical claims. The low frequency of formal reasoning here (36.67%) indicates that while students can apply examples to solve problems, few leverages them to explore the theorem's underlying structure.

To foster deeper epistemic engagement, instruction should explicitly connect example-based practice to generalization. Tasks that prompt students to design their own test cases or compare consistent vs. inconsistent patterns can help them move from using examples as practice tools to viewing them as evidence for mathematical truth (Bariyah & Prabawanto, 2024).

Why do we Need to Test/Verify the Pythagorean Theorem?

The analysis of students' actual calculations in Part 1 revealed a high proficiency in the application of the formula $c^2 = a^2 + b^2$. Out of the 30 participants, 85% successfully performed the squaring and summation of the legs. The most common errors were observed in finding the square root of the sum, particularly for non-primitive triples. This computational data provides a baseline confirming that the students' subsequent observations on parity were grounded in correct mathematical operations.

The prevalence of formal reasoning (66.67%) in responses to this question signals a mature epistemological orientation toward mathematical validation. Students framing verification as a means to confirm universality, reveal patterns, and build understanding align with Schommer's (1990) characterization of learners who view knowledge as complex and evidence-based. Those focusing on real-world application reflect practical epistemology, emphasizing the theorem's utility—a balance that supports both conceptual and procedural fluency.

This finding contrasts with prior studies noting that many students treat mathematical rules as unchanging authority (Fadillah & Rifki, 2024). The presence of formal reasoning here suggests that targeted validation activities can help students shift from passive acceptance to active justification of mathematical principles. However, the small subset referencing skill development alone indicates that some learners still view verification as a procedural exercise rather than an epistemological practice.

Instructional designs that emphasize the purpose of verification—such as exploring historical proofs or examining cases where the theorem does not apply (non-right triangles) can reinforce the role of testing in constructing mathematical knowledge. Facilitating discussions about why validation matters for both theory and practice can help students integrate practical and formal epistemological perspectives (Sandra, 2025).

Why do we Need to Notice Parity Patterns of the Sides' Length?

The exclusive focus on formal reasoning here demonstrates strong recognition of how pattern analysis supports theoretical understanding. Students emphasizing pattern discovery, theorem comprehension, limitation identification, and critical thinking align with constructivist principles that frame learning as active pattern recognition (Piaget, 1972). This ability to analyze data for meaningful relationships reflects epistemological growth toward viewing mathematics as a systematic discipline.

However, the subsequent generalization task reveals a disconnect between pattern awareness and accurate conclusion-making a finding consistent with research noting that students often misinterpret empirical results (Taamneh & Díez Palomar, 2024). While students could identify parity trends, many incorrectly generalized that the theorem fails for even-even or odd-odd leg pairs, indicating epistemic fragmentation where observations are not coordinated with theoretical knowledge.

To address this, instruction should explicitly link pattern observation to logical reasoning. For example, guiding students to recheck calculations, explore counterexamples, or derive why parity combinations affect results (e.g., properties of even/odd squares) can help them reconcile empirical patterns with the theorem's universal validity. Incorporating prediction-observation-explanation cycles can also help students identify and correct misconceptions about parity (Sin, 2014).

Why do Students' Generalizations About Parity Patterns Reflect Inconsistent Reasoning?

The majority of students correctly concluded the theorem works for valid sets. However, they observed that it fails specifically for the set containing odd-odd-odd values (Item 4), which is mathematically invalid. This pattern reflects epistemic reliance on empirical evidence, where students base their conclusions strictly on the results of their calculations rather than generalizing purely from theoretical rules (Lising & Elby, 2005).

Similar to findings by Hutapea et al. (2015), these misconceptions stem from procedural errors or overgeneralization of specific cases. The small subset correctly linking validity to mixed parity suggests that some students can integrate pattern analysis with theoretical knowledge, but most struggle to reconcile discrepancies between their observations and the theorem's universality.

From an instructional standpoint, deliberately surfacing these misconceptions and guiding students to investigate root causes (e.g., miscalculating hypotenuse values, misapplying the formula) can help them develop coherent epistemic reasoning. Tasks that require students to justify their generalizations with multiple examples or derive the theorem's parity properties algebraically can bridge the gap between empirical observation and formal proof (Rahmah & Nuryani, 2024). Additionally, collaborative discussions where students compare conclusions and resolve contradictions can support social mediation of epistemological growth (Vygotsky, 1978).

Limitations of the Study

While the findings provide valuable insights into students' mathematical epistemology, several limitations must be considered. First, the study utilized a small sample size ($n = 30$) from a single public national high school, which limits the generalizability of the results to larger or more diverse populations. Second, the lack of data triangulation, such as the absence of follow-up interviews or classroom observations, means the analysis relied solely on students' written reflections. This may not capture the full complexity of their reasoning processes.

Furthermore, the intentional inclusion of non-right triangle sets as distractors, while useful for testing exclusionary logic, may have introduced instructional-induced biases that influenced student generalizations. Future research should expand the participant pool and incorporate multiple data sources, including verbal protocols and longitudinal tracking, to further validate the observed epistemic transitions.

CONCLUSION

This investigation explored the layers of mathematical epistemology among students as they validated the Pythagorean theorem. By analyzing their responses across visualization, verification, and pattern-seeking tasks, we found that students' epistemic stances are not static but are deeply influenced by how a task is presented. While a practical lens rooted in empirical drawings and real-world utility was evident at the start, a

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shift toward formal reasoning became visible when students were forced to confront the validity of specific parity sets.

A key takeaway from this work is the students' inclination to build heuristic rules from numerical patterns. By design, we included non-right triangle sets as distractors, and this revealed a crucial finding: students often use these 'failing' results to draw boundaries, though they sometimes over-generalize these patterns into restrictive rules (like the mixed-parity bias). This shouldn't be dismissed as a simple error. Instead, it represents a transitional phase in their thinking a moment where they are trying to reconcile what they see in the numbers with the formal laws of geometry.

Ultimately, these results point to the necessity of more deliberate task designs in the classroom. If we want students to move beyond simple pattern-matching and toward robust mathematical proof, our instructional examples must be carefully curated. By treating non-right triangles as essential 'boundary markers' rather than mere distractors, educators can help students develop a more sophisticated and universal understanding of mathematical truths.

Consent and Ethical Approval:

As per international standards and institutional guidelines, ethical approval for this study was obtained prior to data collection. Informed consent was secured from students' parents or legal guardians, and assent was obtained from the participating students themselves.

All participants were assured of confidentiality and anonymity, with data stored and processed in accordance with privacy regulations. Participation was voluntary, and students were informed they could withdraw from the study at any time without penalty or impact on their academic standing. All data collected has been organized and preserved by the researchers for potential future analysis, while maintaining strict safeguards to protect participant identities.

Competing Interests Disclaimer:

Authors have declared that they have no known competing financial interests OR non-financial interests OR personal relationships that could have appeared to influence the work reported in this paper.

Disclaimer (Artificial intelligence)

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc.) and text-to-image generators have been used during the writing or editing of this manuscript.

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APPENDIX

Students Mathematical Epistemology in Validating Pythagorean Theorem

(SMEVPT) Questionnaire

Name: ____Date: _________________

PART 1

<u>**Directions:**</u> The task shown below requires you to verify the relation $c^2 = a^2 + b^2$ (Pythagorean theorem) using the assign values of sides' length of the right triangle. Make sure that your answers are written clearly and legibly on the space provided for each number 1 to 8. Use the example as your guide.

Test/verify the relation $c^2 = a^2 + b^2$ (Pythagorean theorem) using the assign values of sides' length of the right triangle		**Observations/Notices** **Instruction:** In order to be organized in your observations/notices, please answer the following guide questions:			**Generalization/** **Conclusion**
		Mathematical verification ($c^2 = a^2 + b^2$)	Classification	Parity Variation	
Examples:	1) Let $a = 6$, $b = 8$ and	$10^2 \overset{?}{\underset{?}{=}} 6^2 + 8$	Valid		When the parity (even, odd) of the

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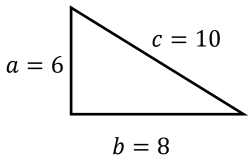
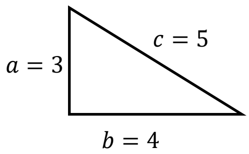
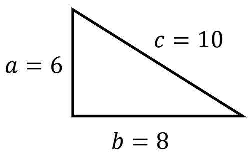
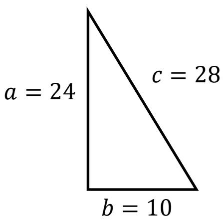
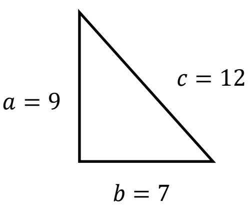
	$c = 10$ 	$100 = 36 + 64$ $100 = 100$ (satisfied)		$a = 6,\ b = 8$ and $c = 10$ even even even	legs of the right triangle are all even then it will satisfy the relation.
	2) Let $a = 3$, $b = 4$ and $c = 5$ 	$5^2 \overset{?}{\underset{?}{=}} 3^2 + 4^2$ $25 = 9 + 16$ $25 = 25$ (Satisfied)	Valid	$a = 3, b = 4$ and $c = 5$ odd even odd	When the parity (even, odd) of the legs of the right triangle are of opposite parity then it will also satisfy the relation.
1) Let $a = 6, b = 8$ and $c = 10$ 			Valid		
2) Let $a = 24, b = 10$ and $c = 28$ 			Invalid		
3) Let $a = 9, b = 7$ and $c = 12$ 			Invalid		

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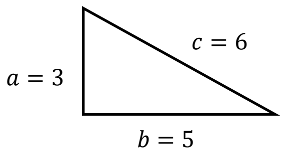
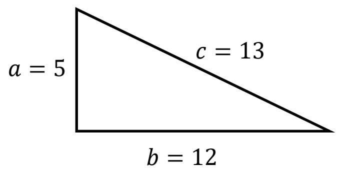
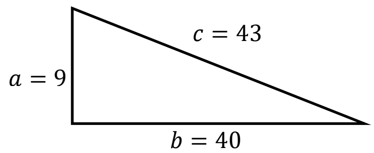
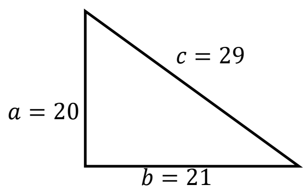
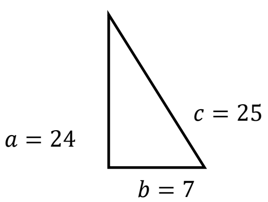
ISSN No. 2454-6186 | DOI: 10.47772/IJRISS | Volume X Issue IV April 2026

4) Let $a = 3$, $b = 5$ and $c = 6$  		Invalid		
5) Let $a = 5$, $b = 12$ and $c = 13$  		Valid		
6) Let $a = 9$, $b = 40$ and $c = 43$  		Invalid		
7) Let $a = 20$, $b = 21$ and $c = 29$  		Valid		
8) Let $a = 24$, $b = 7$ and $c = 25$  		Valid		

PART 2

Open-Ended Questionnaire

INSTRUCTION: Answer the following questions using the language (English-Maranaw-Tagalog) you are most convenient with.

1. Why do we need to draw/illustrate the right triangle with assign values for side's length?

____

2. Why do we need to make use of specific examples of assign values of the sides' length of the right triangle to test the relation $c^2 = a^2 + b^2$ (Pythagorean theorem)?

____

3. Why do we need to test/verify the mathematical relation $c^2 = a^2 + b^2$ (Pythagorean theorem)?

____

4. Why do we need to take a look/notice the testing/verification results and pattern/trend of parity (even, odd) of the sides' length of the right triangle?

____

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5. Construction of generalization/conclusion. Base from the testing/verification results and pattern/trend of parity (even, odd) of the sides' length of the right triangle, construct/write one sentence only generalizing the testing/verification results and pattern/trend of parity (even, odd) of the sides' length of the right triangle for the pair numbers 1 and 2, 3 and 4, 5 and 6, and 7 and 8. Write them in enumerated form.

   (For pair numbers 1 and 2)

   (For pair numbers 3 and4)

   (For pair numbers 5 and 6)

   (For pair numbers 7 and 8)